OFDM Transmission Corrupted by Impulsive Noise

Jürgen Häring, Han Vinck

University of Essen
Institute for Experimental Mathematics
Ellernstr. 29
45226 Essen, Germany

e-mail: haering@exp-math.uni-essen.de

Abstract: OFDM is a promising technique being used for bandwidth efficient communication over the power-line channel. It provides excellent possibilities to adapt to the frequency-selectivity of the channel. The influence of impulsive noise on the OFDM transmission is not well analyzed yet. We choose Middletons Class A man-made noise model to statistically describe the impulsive interference and study the capacity of the additive impulsive noise channel. With the obtained results we will discuss the loss in performance, if the conventional OFDM-receiver designed for Gaussian noise is used for reception on such a channel. In the second part we describe a new iterative algorithm suited to suppress impulse-like interference.

KEYWORDS: power-line, OFDM, impulsive noise, Class A noise, channel capacity

1. Introduction

It is well known that the data transmission over power-lines provides many attractive properties. A mayor drawback of the technology is the difficult transmission channel: the transmitted data is not only corrupted by colored Gaussian noise, but also by different types of narrowband interference and impulsive noise. Especially in the high frequency domain the attenuation additionally becomes highly frequency selective [3] [4].

One promising modulation scheme for data transmission over power-lines is OFDM. It provides excellent possibilities to handle the colored noise, the narrowband interference and the frequency selective attenuation of the channel. The behavior of the OFDM transmission corrupted by impulsive noise was not well analyzed yet. Some authors argue that impulse-like interference occurring in the time domain gets suppressed after demodulation (in the frequency domain) by spreading the impulses over a large number of carriers. Applying the central limit theorem, they treat the noise as an additional Gaussian component after demodulation and use detection schemes designed for the Gaussian channel [5].

The goal of our paper is to give more insight into the influence of impulsive noise on OFDM. In the first section we introduce the selected impulsive noise model and give a brief review of OFDM. In the second part we will discuss our results on the capacity of the impulsive noise channel. This gives us the opportunity to compare the performance of the optimally designed receiver to the performance of the conventional OFDM receiver, both used in the impulsive noise environment. By conventional receiver we mean the receiver designed for the additive Gaussian noise channel. In the third part we describe a new algorithm suited to significantly mitigate the impulsive noise interference on the OFDM transmission.

2. Impulsive Noise Channel Model and OFDM

In order to analyze the performance of OFDM corrupted by impulsive noise a statistical model for the interference is required. In this paper we choose Middletons canonical man-made noise model [7]. It comprises the influence of impulsive noise and an additionally Gaussian component. The model is known to fit very well to a broad spectrum of measured data [7].

The starting point for deriving the impulsive noise model is the assumption that a large number of statistical independent interferers contribute to the noise. According to the bandwidth of the noise emitted by each of the interferers, Middleton classifies the noise in three general classes, class A, B and C. In this paper we focus on the Class A noise model. Here, the noise bandwidth is assumed to be comparable or less than the bandwidth of the disturbed communication system and so transient effects in the analog receiver stages can be neglected. Additionally, a Gaussian noise component is added to model the (almost) always present thermal receiver noise. The Class A noise probability density function is given by [7].
\[ p_n(\eta) = e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m!2\pi\sigma^2_m} \exp\left(-\frac{\eta^* \eta}{2\sigma^2_m}\right) \]

with \( \eta^* \) denoting the complex conjugate of \( \eta \) and

\[ \sigma^2_m = \frac{A + T}{1 + T}. \]

The parameter \( A \) is called impulsive index given by the product of the average number of impulses per unit time and the mean duration of the emitted impulses entering the receiver. For \( A \to \infty \) the noise gets Gaussian distributed, for small \( A \) the noise gets more structured/impulsive. The parameter \( T \) gives the ratio between the mean power of the Gaussian and the mean power of the impulsive noise component.

Besides the given statistical description, we need a channel model for our further investigations. As we are interested in the influence of impulsive noise on the OFDM transmission in general we keep the channel model as simple as possible. We will therefore study the additive impulsive noise channel in this paper:

\[ r = s + n \]

with the notation \( s \) for the transmitted symbol, \( n \) for the Class A distributed random variable and \( r \) for the received value. Usually there is no need to use an OFDM modulation on such a channel, but as we are interested in understanding the basic principles this simple setup is well suited for our studies. Throughout our paper we use the following notations and specifications for our transmission scheme:

- we investigate a K-carrier OFDM system, \( K \) is assumed to be large
- we use a M-QAM signal constellation on each carrier
- time domain vectors are denoted by small, frequency domain vectors by capital letters
- each component of the frequency domain vectors (e.g. \( S = (S_0, S_1, \ldots) \)) is a complex number representing a signal point of a QAM-modulation scheme
- the sender modulates the information vector \( S \) by performing the inverse Fourier transform: \( s = F^{-1}S \) with the Fourier-matrix \( F \)
- according to the channel model, the receiver gets a noisy version of the transmitted data: \( r = s + n \). For demodulation the receiver computes the Fourier transform: \( R = Fr = S + N \). In the impulsive noise case the random variables in \( N \) are statistical dependent after performing the transform. This can be proofed by calculating the linear transform \( N = Fn \).

We have now defined our channel model and the communication system setup. We will use this in the following sections to investigate the channel capacity of the impulsive noise channel and to describe our new noise mitigation algorithm for OFDM transmission.

### 3. Channel Capacity

In this section the channel capacity of Middletons memoryless additive Class A impulsive noise channel with an average energy constraint on the input is studied. The channel capacity gives a general performance bound for reliable communication over a noisy channel [1]. We are interested in comparing the capacity of the Gaussian noise channel with the capacity of the Class A impulsive noise channel. From this investigations we will draw some conclusions according to the performance of an conventional OFDM-receiver used in an impulsive noise environment. We calculate the channel capacity for the memoryless additive noise channel

\[ Y = X + N \]

with average energy constraint on the input and statistically independent random variables \( X \) and \( N \). We denote the channel input as \( X \), the random noise as \( N \), the channel output as \( Y \) and their probability density functions as \( p_X(x) \), \( p_Y(y) \) and \( p_N(n) \). Calculating the channel capacity means that for a given noise distribution \( p_N(n) \) the input distribution \( p_X(x) \) has to be found which maximizes the so called mutual information [1]

\[ C = \max_{p_X(x)} \left( H(Y) - H(N) \right) \]

with the entropy function \( H(.) \). Unfortunately, it is not possible to find an explicit expression for the optimum input distribution needed. We therefore have to fall back on lower and upper bounds for the channel capacity. We calculate three different bounds by making the following assumptions:
1. Gaussian-Output: For calculating this bound it is assumed that a Gaussian distribution can be achieved at the channel output. For average energy constraint input signals and given noise statistics this assumption maximizes $H(Y)$ and therefore also the capacity $C$. Hence, the Gaussian-Output bound is an upper bound.

2. Gaussian-Input: Here the mutual information of the Class A channel for a Gaussian input distribution is calculated. As we chose one arbitrary input distribution, this is of course a lower bound.

3. Gaussian-Part: The impulsive-noise part in the Class A model is neglected so only the Gaussian part contributes to the disturbance. As we reduced the noise interference, this is an upper bound.

The different bounds are depicted in Fig. 1 for a highly impulsive noise environment. Additionally, the capacity curve for the AWGN channel is shown.

![Figure 1: Upper and lower bounds on Middletons additive Class A noise channel](image)

From the Figure mainly two points can be learned:

1. The lower bound generated by using a Gaussian input distribution for the channel is very close to the upper bounds and can not even be distinguished from them in some regions. This means that the lower bound is a very good approximation to the real channel capacity (at least for these values of $A$ and $T$) and that taking a Gaussian input distribution is a good choice. For a large number of carriers the OFDM-modulation generates a Gaussian distributed signal. According to this aspect we conclude that OFDM is well suited for the transmission over the additive Class A channel.

2. Many authors argue that impulsive noise is no problem because the impulses get spread over a large number of carriers and get therefore suppressed [5]. Applying the central limit theorem they treat these impulses as an additional Gaussian component. If we follow this argumentation from the standpoint of channel capacity we have to study a super-channel consisting of the combination of impulsive noise channel and the OFDM modulation scheme, where the statistical dependencies of the noise on the different carriers is neglected. For infinitely many carriers the result is an AWGN-channel with mean noise energy equal to the mean impulsive noise energy. The capacity curve of this channel is also shown in Fig. 1. It can be seen that for the given example performance degradations of up to 35dB occur. From this we conclude that a special treatment of the impulsive noise is necessary.

4. Iterative Noise Suppression Algorithm

As seen in the previous section there are two classical ways of detecting OFDM-Symbols corrupted by impulsive noise. In the first case the detection is done independently for each subcarrier and the optimum detector is derived based on the marginal probability density function for this carrier. Usually the optimum detector for the
Gaussian channel is used. The result of this method is a huge performance degradation, because the statistical dependence between the noise on the different carriers is neglected. In the second case maximum likelihood detection in the time or in the frequency domain is used. In the time domain all components of the vector $S$ are statistically dependent and in the frequency domain all components of the noise vector $N$ are statistically dependent. To perform a maximum likelihood detection the receiver has to investigate all possible sequences for $S$. This results, at least in the practical situation, in a problem with insuperable computational complexity.

We therefore introduce an iterative algorithm which exploits the nature of the impulsive noise and drastically increases the performance of some OFDM transmission schemes. In [2] a related algorithm was proposed to reduce the influence of clipping noise introduced by hard limiting the peaks of an OFDM-transmission. This algorithm is not suited for correcting impulsive noise errors. The major difference between the two algorithms is that we approach the problem from a different perspective: we do not try to give a good estimate for the sequence $S$ that was send given the received vector $R$, but we try to find a good estimate for the noise vector $N$ given $R$. This leads us to modified decision rules. The proposed algorithm is described in the following. We use the notation given in the section about OFDM and additionally use superscript letters to denote the number of iteration we are in. The return value of the algorithm is $\tilde{S}$ as an estimate for the sequence $S$ that was sent.

$$r^{(0)} = r$$
$$l = 0$$

repeat until stop - criterion valid

$$R^{(l)} = F r^{(l)}$$
$$S^{(l)} = \text{detect}(R^{(l)})$$
$$N^{(l)} = R - S^{(l)}$$
$$n^{(l)} = F^{-1} N^{(l)}$$
$$n^{\text{estimate}} = \text{estimate}(n^{(l)})$$

$$r^{(l+1)} = r - n^{\text{estimate}}$$
$$l = l + 1$$

end repeat

$$R^{(l)} = F r^{(l)}$$
$$\tilde{S} = \text{detect}(R^{(l)})$$

In the following we discuss the different elements of the algorithm:

**Stop-criterion**: The algorithm is terminated, if $S^{(l)} = S^{(l+1)}$ is valid for the first time. It can easily be shown that any further iteration does not change the result of the algorithm. The iteration loop is also terminated, if a maximum number of iterations is exceeded.

**Detect-operator**: The operator investigates each carrier independently in the frequency domain and maps each given $R_i^{(l)}$ on a signal point in the QAM-constellation. We use the same detector as in the Gaussian noise case and therefore choose the signal-point $S_i^{(l)}$ with the minimum Euclidean distance to the given value $R_i^{(l)}$. Of course, in general there is no need to take a hard decision rule here.

**Estimation-operator**: There are several options for the determination of $n^{\text{estimate}}$. Minimizing the quadratic error for the estimation or performing a MAP-estimate are two possibilities leading to more general algorithms. In those cases the estimators have to be adaptive to the current values of $n^{(l)}$ and $l$ and the statistical properties of $n^{(l)}$ have to be known. The algorithm then gets highly sensitive to errors in those assumptions. As we are interested in the basic principle of the algorithm, we will discuss a simple version and use the following, nonadaptive estimation-operator:

$$\text{estimate}(n^{(l)}) = \begin{cases} 
0 & \text{for } n^{(l)} < T \\
 n^{(l)} & \text{for } n^{(l)} \geq T 
\end{cases}$$
with some threshold $T$. To see that this estimation makes sense, we have to discuss the statistical properties of $n^{(0)}$ first. In our presentation we will show that at least the first noise-estimate $n^{(0)}$ can be approximated by

$$n^{(0)} = n + e,$$

where $n$ is the original noise-vector occurred on the channel, $e$ is an additional, statistical independent noise vector and $\gamma$ a factor smaller than one. The contribution of $e$ is caused by wrong decisions made by the detect-operator. In the low SNR-region a lot of wrong decisions are made and superimposed by the inverse Fourier transform following the detect-operator in the algorithm. So $e$ can be assumed to be Gaussian distributed. In the vector $n^{(0)}$ the high noise peaks caused by the impulsive noise vector $n$ can now be distinguished from the underlying Gaussian noise. In the high SNR-range error events are rare compared to the number of carriers and so the assumption of $e$ being Gaussian distributed is not valid anymore. In Fig. 2 we give the calculated variance of $e$.

![Figure 2: Calculated variance $\sigma_e^2$ of the "decision" noise](image)

For our algorithm we choose the threshold $T$ using the variance $\sigma_e^2$. As the assumption of $e$ being Gaussian distributed gets worse in the high-SNR domain we modify our threshold in this region and set it to a constant value. The threshold is given by:

$$T_{SNR} = 3 \cdot \max(\sigma_{e,SNR}; \sigma_{e,SNR=7dB}).$$

The subscript is used to denote that the threshold $T_{SNR}$ is a function of the SNR on the channel. In Fig. 3 some simulation results are given for a 1024-carrier OFDM system. The different curves show the error rates after the respective number of iterations. After seven iterations no error events were observed anymore. Without evidence we want to note here that more sophisticated estimation-operators achieve a faster convergence of the algorithm.
1024-Carrier OFDM Transmission in Class A Noise: $A=0.1$, $T=1e^{-4}$

![Graph showing performance of iterative noise reduction algorithms after different iterations.]

Figure 3: Performance of the iterative noise reduction algorithms after a different number of iterations.

**Conclusion**

In this paper we analyzed the performance of the OFDM transmission scheme corrupted by impulsive noise. Using channel capacity arguments we showed, that using the conventional Gaussian noise OFDM receiver in an impulsive noise environment results in strong performance degradations. In the second part of our paper we described a new iterative algorithm suited for mitigating the influence of the impulsive noise on the OFDM transmission.

**References**


