Coding and Modulation for Power-Line Communications

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Abstract: We describe a transmission scheme combining 4-FSK modulation with diversity and coding to make the transmission over power lines robust against permanent frequency disturbances and impulse noise. We use simple non-coherent detection and derive the corresponding error correcting capabilities of the scheme. Since we use 4-FSK, the scheme is in agreement with the existing CENELEC norms. The scheme can be considered as a form of coded Frequency Hopping and is easy extendable to any frequency range.

keywords: modulation; power-line communications; coding.

1. INTRODUCTION

Modulation schemes with a constant envelope signal modulation such as binary-FSK and M-ary FSK are in agreement with the CENELEC norms, EN 50065.1, part 6.3.2. We focus on the low frequency range below 150 kHz. In this region, there are several channel characteristics like attenuation, permanent frequency disturbances and impulsive noise that need special attention. The main goal of the contribution is to show that a combination of M-ary FSK modulation and coding can provide for a constant envelope modulation signal, frequency spreading to avoid bad parts of the frequency spectrum, and time spreading to facilitate correction of frequency disturbances and impulse noise simultaneously.

Channel characteristics regarding attenuation and noise have been reported in [2] and [3]. Television sets or computer terminals generate narrow band noise and thus, this type of noise is permanent over a long period of time and of great importance in power line communication systems. Impulse noise has been reported in [3]. From [3] it can be concluded that impulses have a duration of typically less than 100 μsec. Measurements in networks indicate that the inter-arrival times are independent and .1 to 1 second apart.

In an M-ary FSK modulation scheme, symbols are modulated as one of the sinusoidal waves described by

\[ s(t) = \sqrt{\frac{2E_s}{T_s}} \cos(2\pi f_1 t); \quad 0 \leq t \leq T_s; \quad f_1 = f_0 + \frac{i-1}{T_s}, 1 \leq i \leq M. \]  

where \( i = 1,2,\ldots,M \) and \( E_s \) is the signal energy per modulated symbol. The signals are orthogonal and for non-coherent reception the frequencies are spaced by \( 1/T_s \) Hz, being the transmission rate. To avoid abrupt switching from one frequency to another, the information bearing signal may modulate a single carrier whose frequency is changed continuously (CPFSK). The demodulation may be accomplished using \( 2M \) correlators, 2 per signal waveform. The optimum non-coherent demodulator detects \( M \) envelopes and outputs as estimate for the transmitted frequency the one that corresponds to the largest envelope. This makes the demodulation process simple and independent from the channel attenuation. However, there are several channel disturbances that degrades the performance:

- non-coherent demodulation with largest envelope detection is not optimum in case of a simplified model of a frequency selective channel as is the power line channel, see [1];
- narrow band noise may cause large envelopes and thus errors to occur at the demodulator output;
- impulse noise has a broad band character and thus could lead to a multiple of large envelopes.

To be able to handle these types of noise processes, we propose to modify the demodulator in such a way that the detected envelopes can be used in the decoding process of a special class of error control codes, called permutation codes. The combination of M-FSK with permutation coding gives rise to a constant modulator output envelope and includes frequency- and time diversity. Frequency- and time diversity can be expected to give robustness against narrow band noise and impulse noise. We first describe the properties of the combined modulation/coding scheme. Then we give the modification of the demodulator and the influence of the channel noise on the demodulator output.
We error correcting capabilities for the detection/decoding scheme are treated for background noise, impulse noise and frequency disturbances. The simulations indicate that the minimum distance plays an important role if frequency disturbances or impulse noise is present. The scheme that is optimum for background noise is worst for the other disturbances.

2. COMBINED MODULATION and CODING

Consider an M-ary FSK signal set that consists of an orthogonal set of M frequency-shifted signals. We use the integers 1,2,...,M to represent the M frequencies, i.e. the integer i represents \( f_i \). A message is encoded as a code word of length M with the integers 1,2,...,M as symbols, where every symbol occurs only once in the code word. The symbols of a code word are transmitted in time as the corresponding frequencies and thus the transmitted signal has a constant envelope output. Let \(|C|\) denote the cardinality of the code.

**Definition.** A permutation code \( C \) consists of \(|C|\) code words of length M, where every code word contains the M different integers 1,2,...,M as symbols.

| TABLE 1 |
|-----------------|-----------------|-----------------|
|                 | \( d_{\text{min}} = 4 \) | \( d_{\text{min}} = 3 \) |
| message | code word | message | code word | message | code word |
| 1 | 1, 2, 3, 4 | 1 | 1, 2, 3, 4 | 7 | 4, 2, 1, 3 |
| 2 | 2, 1, 4, 3 | 2 | 1, 3, 4, 2 | 8 | 4, 3, 2, 1 |
| 3 | 3, 4, 1, 2 | 3 | 2, 1, 4, 3 | 9 | 1, 4, 2, 3 |
| 4 | 4, 3, 2, 1 | 4 | 2, 4, 3, 1 | 10 | 2, 3, 1, 4 |
| 5 | 3, 1, 2, 4 | 5 | 3, 1, 2, 4 | 11 | 3, 2, 4, 1 |
| 6 | 3, 4, 1, 2 | 6 | 3, 4, 1, 2 | 12 | 4, 1, 3, 2 |

The codes as given in table 1 each have \( M=4 \) different numbers in every code word and a minimum difference between any two code words or minimum Hamming distance \( d_{\text{min}} \) of 4 and 3, respectively. As an example, message 3 is transmitted in time as the series of frequencies \( (f_3, f_4, f_1, f_2) \). It has been shown, [5], that the required bandwidth and symbol duration time of permutation encoded M-FSK are given by

\[
B = M \cdot \frac{b \cdot M}{\log_2 |C|} , \quad T_s = \frac{M}{B} ,
\]

(2)

where the information transmission rate is \( b \) bits per second, and \(|C|\) the number of code words in a code. To maximize the efficiency, we have to find the largest \(|C|\) for a given \( M \) and \( d_{\text{min}} \). It is easy to see that for a code with code words of length \( M \) each having \( M \) different numbers \( d_{\text{min}} \) is always \( \geq 2 \). The cardinality \(|C|\) of this code is \( M! \). In [6], we showed that the cardinality of the permutation codes is upper bounded by

\[
|C| \leq \frac{M!}{(d_{\text{min}} - 1)!} ,
\]

(3)

In the next section we discuss the modification of the demodulator and the use of the modified demodulator output in the decoding of permutation codes.
3. DEMODULATION and DECODING
We first briefly recall the conventional non-coherent demodulator and its modification as given in [1]. The demodulation may be accomplished using 2M correlators, 2 per signal waveform.

- The sub-optimum non-coherent demodulator computes M envelopes and outputs as estimate for the transmitted frequency the one that corresponds to the largest envelope. See Fig. 1 for a general envelope detector.
- An optimum decision rule can be derived using the knowledge of the SNR per sub-channel for a particular frequency. In practical schemes the SNR per sub-channel can be obtained with the help of a well defined preamble or by using the output of the correlators, see also [1].

![Diagram of envelope detector for frequency \( f_k \).](image)

The output \( r_k \) can be normalized with respect to the noise variance \( \sigma_k^2 \). The probability density function for the normalized output \( y_k := r_k / \sigma_k \) at time \( j \) is given by [3, 4]

\[
\Delta_{k,j} := p(y_k | \text{frequency } k \text{ transmitted}) = y_k \exp(-\frac{y_k^2 + 2E_k / \sigma_k^2}{2}) I_0(y_k \sqrt{2E_k / \sigma_k^2}),
\]

\[
\mathcal{V}_{k,j} := p(y_k | \text{frequency } k \text{ not transmitted}) = y_k \exp(-\frac{y_k^2}{2}),
\]

where \( E_k \) and \( \sigma_k^2 \) are the received symbol energy and the noise variance for the particular channel \( k \), respectively and \( I_0 \) is the modified Bessel function of order zero. For the permutation code, we know that a particular frequency must be present only once per code word. This fact can be used to estimate received symbol energy as well as the noise variance per sub-channel. We assume that we have these values available at the receiver.

We define 4 types of detector/decoder combinations. For that purpose we use the \( M \times M \) matrix \( Y \).

1. The classical detector with column wise hard decisions. In the matrix \( Y \) the element

\[
(i, j) = 1 \text{ if } y_i \text{ is the largest envelope detector output at time } j; \text{ otherwise } (i, j) = 0.
\]

The value \((i, j) = 1\) corresponds to the assumption that frequency \( f_i \) has been transmitted. Hence, the permutation decoder compares its code words with the corresponding frequencies and outputs the code word at minimum distance.
2. The \textit{modified classical} detector with column wise soft decisions. In the matrix $Y$ the element

$$(i, j) = 1 \text{ if } \Delta_i \text{ is the largest density at time } j; \text{ otherwise } (i, j) = 0.$$  

Again, the permutation decoder compares its code words with the corresponding frequencies and outputs the code word at minimum distance.

3. The column and row wise \textit{hard-decision threshold detector}. In this case we use a threshold $T_i$ for every envelope detector. The position of the threshold values $T_i$ can be optimized depending on $E_i$ and $\sigma_i^2$. A practical value could be $0.6 \sqrt{E_i}$. The elements for the column $j$ are given by

$$(i, j) = 1 \text{ if } y_i > T_i; \text{ otherwise } (i, j) = 0.$$  

The permutation decoder compares its code words with the corresponding frequencies and outputs the code word at minimum distance.

4. The column and row wise \textit{soft-decisions threshold detector}. We now put in the elements of $Y$ the values

$$(i, j) = \frac{\Delta_{i,j}}{\sqrt{V_{i,j}}} \text{ for all } i \text{ and } j.$$  

The permutation decoder computes for a particular code word $k$ with a frequency $f_i$ at time $j$, the value

$$F_k = \frac{M}{\prod_{i,j=1}^{n} \sqrt{V_{i,j}}}; \quad k = 1, 2, \ldots, |C|.$$  

It can be shown that the Maximum Aposteriori Probability (MAP) demodulator outputs the value of $k$ that maximizes $F_k$. In fact, we calculate the normalized probability that we receive a certain matrix $Y$ given a code word $k$. For optimum performance, we need to know $E_i$ and $\sigma_i$.

4. ERROR CORRECTING PROPERTIES

For the detector/decoder 1-3, we output the message with the minimum number of differences with the entries in the matrix $Y$. For detector 4, we output the MAP code word. Since detector 3 is very simple to implement, we further analyze the error correcting capabilities for this type of threshold detection.

\textbf{Permanent narrow band noise:} For the example with $M = 4$ and $d_{\text{min}} = 4$, a permanent disturbance (narrow band noise) present at the sub-channel for frequency $f_4$ and transmission of code word $(3, 4, 1, 2)$ could lead to a matrix $Y$

$$
\begin{array}{cccc}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 \\
\end{array}
$$

The decoder compares the demodulator output with all possible transmitted code words. It outputs the code word for which the maximum number of agreements with the symbols at the demodulator output occurs. For the example, all symbols corresponding to code word 3 are present and thus correct decoding follows. Since code words are different in at least $d_{\text{min}}$ positions, $d_{\text{min}} - 1$ errors of this type still allow correct decoding. The example code has $d_{\text{min}} = 4$ and hence, we can tolerate the presence of 3 permanent disturbances present in the demodulator output.

\textbf{Impulse noise:} For a signaling scheme using a signaling rate of 10kHz, we have a symbol duration of 100\,\mu s, which is in the range of the typical impulse duration of 100\,\mu s. So, impulse noise, affecting at least two adjacent symbols cannot be excluded. Due to the broad frequency band character, impulse noise may cause the demodulator to output the presence of all frequencies. This type of noise can be seen as erasures. Hence, two affected adjacent
transmissions may reduce the minimum distance of the code by 2. At a signaling rate of 10 kHz, a code with \(d_{\text{min}} = 3\) is thus capable of correcting 2 permanent disturbances or to give a correct output in the presence of an impulse. For higher signaling rates, more symbols may be affected and thus a code with a larger minimum distance is needed. Suppose that we transmit the example code word \(\{3, 4, 1, 2\}\). If an impulse noise causes all envelopes to be present at two symbol transmissions, then we may have as a matrix

\[
Y = \begin{pmatrix}
0 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 \\
1 & 1 & 1 & 0 \\
0 & 1 & 1 & 0 
\end{pmatrix}
\]

Comparing this output with the possible transmitted code words gives a difference (distance) of zero to the correct code word and two to all other code words. Even if three of these multi-valued outputs occur, we are still able to find back the correct code word since there is always a remaining symbol that gives a difference of one to the incorrect code words.

**Background noise** degrades performance by introducing unwanted (called insertions) demodulator outputs or by causing the absence (called deletion) of a transmitted frequency in the demodulator output. Note that for this type of "threshold" demodulation, the decoding is still correct for \(d_{\text{min}} - 1\) errors of the insertion/deletion type.

- The absence of a frequency in the demodulator output always reduces the number of agreements between a transmitted code word and the received code word by one. The same is true for the other code words having the same symbol at the same position. If the symbols are different the number of agreements does not change.
- The appearance of every unwanted output symbol may increase the number of agreements between a wrong code word and the received code word by one. It does not decrease the number of agreements between a transmitted code word and the received code word.

In conclusion, we can say that the introduced thresholds in the modified demodulator in combination with the permutation code allow the correction of \(d_{\text{min}} - 1\) incorrect demodulator outputs caused by narrow band-, impulse- or background noise. In Table 2, we summarize the expected decoding properties for the different detectors and different types of noise. These conclusions also follow from the simulation results as presented in Figs. 4-6.

### TABLE 2

<table>
<thead>
<tr>
<th>noise type</th>
<th>detector type</th>
<th>background noise</th>
<th>impulse noise</th>
<th>frequency jammer</th>
</tr>
</thead>
<tbody>
<tr>
<td>classical</td>
<td>classical</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>classical +</td>
<td>classical +</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>threshold</td>
<td>threshold</td>
<td>0</td>
<td>+ +</td>
<td>+</td>
</tr>
<tr>
<td>MAP</td>
<td>MAP</td>
<td>0</td>
<td>- -</td>
<td>- -</td>
</tr>
</tbody>
</table>

5. SIMULATIONS

It can be shown [1] that the probability of an insertion or deletion error due to background noise is of the same type as for simple Amplitude Shift Keying (ASK). In fact, the bit error probability for the indicated threshold setting is approximated as \(P_{e, \text{ASK}} \approx 0.5 \exp(-\text{SNR}_i / 4)\), where \(\text{SNR}_i = E_i/N_i\) is the SNR for channel \(i\) and \(N_i\) is the single
sided noise power spectral density for sub-channel i. From [2] it can be concluded that the received signal power $S_r$ for a bad transmission channel and the worst case single sided noise power spectral density are given by

$$S_r = S_m \cdot 10^{-0.01 \cdot L}, \quad N_{te, \text{word}}(f) = 10^{-8-4 \cdot 10^{-5} \cdot f},$$

where $S_m$ is the transmitter output power and $L$ the distance between transmitter and receiver in meters. According to the CENELEC norms, we take $S_m = 25$ Watt, and we may use a bandwidth up to 95 kHz. Assuming that the received signal energy is the same for all sub-channels, the received signal energy and received power are related as $E_r = S_r T_r$

According to the CENELEC norms, we take $S_m = 25$ Watt, and we may use a bandwidth up to 95 kHz. Assuming that the received signal energy is the same for all sub-channels, the received signal energy and received power are related as $E_r = S_r T_r$. For a 4-ary FSK scheme with an information rate of $b = 4.8$ kb/s. This scheme has the following properties:

- $B = \frac{4.8 \cdot 16}{\log_2 |C|}$ kHz, divided into 4 sub-bands each of bandwidth $B/4$.

The total power used at a distance of 500 meters (10) is $25 \cdot 10^{-5}$ W. The noise power spectral density (11) at $(95-B)$ kHz is $N_{te} = 10^{-8-0.04(95-B)}$ W/Hz. We then obtain a signal to noise ratio lower bounded by

$$\text{SNR} > \frac{25 \cdot 10^{-5}}{(B/4) \cdot 10^3 \cdot 10^{-8-0.04(95-B)}}.$$

Table 3 gives an overview of the obtained results. We included the uncoded situation where the encoder uses 4 frequencies to transmit the information. For an attenuation of 100 dB per Km, every dB corresponds to 10-meter. In the same way as for AWGN channels we can define coding gain CG as the relation between the coded number of bits per code word and the maximum number of information bits per code word multiplied with $(t+1)$, where $t$ is the number of errors that can be corrected, i.e. for our code

$$\text{CG} = \frac{\log_2 |C| \cdot d_{\text{min}}}{\log_2 (M^M)}.$$

We included this figure in Table 3. If the noise spectral density is flat, then this definition is the same as for the AWGN channel. It can be seen that even for an AWGN channel our example code gives only a small improvement. Here we do not expect any gain since the noise power spectral density has a slope of 4 dB per 10 kHz. Hence, the advantage of using this code is the capability to correct frequency disturbances and impulse noise. From the SNR it follows that we can expect detection errors in the order of $10^{-5}$ at a distance of 800 meters. This can also be seen from the simulation results.

| TABLE 3 |

<table>
<thead>
<tr>
<th>B</th>
<th>SNR</th>
<th>CG in dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.6 kHz</td>
<td>44 dB</td>
<td>0</td>
</tr>
<tr>
<td>16 kHz</td>
<td>40 dB</td>
<td>0.6</td>
</tr>
<tr>
<td>21 kHz</td>
<td>37 dB</td>
<td>1.3</td>
</tr>
<tr>
<td>38 kHz</td>
<td>27 dB</td>
<td>0</td>
</tr>
</tbody>
</table>
The size of the code books depends on the minimum Hamming distance between the code words. Since we fixed the information rate, the signal-to-noise ratio in Table 3 depends on the bandwidth B. For \( d_{\text{min}} = 4 \), the signaling scheme can tolerate up to 3 frequency disturbances and the information rate equals 4.8 kbps. Using (9), one sees that the error probability due to background noise can be neglected and is of no importance at all. For larger values of \( M \), the codes are more efficient and coding gain can be improved. However, the decoding process gets more complex. Another way to improve the coding gain is to use convolutional codes. These codes do not change the rate but artificially increase the length of the code words and the code efficiency. However, as mentioned before, 1 dB in SNR is equivalent to only 10 meters.

An additional option that can be implemented is to consider the transmitted code words as symbols of a longer code. We may for instance use a Reed Solomon (RS) code, see [4]. Symbol (code word) errors are then corrected by the RS code. Of course, the code symbols of the RS code need to be code words of our permutation code. The parameters of such a concatenated coding scheme depend on the particular application.

We now present the simulation results for the above detection/decoding scheme for \( M = 4 \).

- Fig. 1. Threshold and MAP detection for \( d_{\text{min}} = 3 \) and un-coded for background noise only. One clearly sees that the MAP detector is superior but also that coding does not improve performance. This is due to the fact that the SNR is worse by about 6 dB. Together with the coding gain, the overall loss is in the order of 5 dB, or 50 meter.

- Fig. 2. In this figure we give the simulation results for the threshold detector for \( d_{\text{min}} = 2, 3, 4 \) for impulse noise, two frequency disturbances and background noise. Observe that the code with \( d_{\text{min}} = 4 \) still has good performance. The dependency on the distance is clear from the error floors. Furthermore, observe that the MAP receiver for the \( d_{\text{min}} = 4 \) code has the worst performance.

- Fig. 3. This figure compares the un-coded performance with threshold detection for the \( d_{\text{min}} = 3 \) code for background noise combined with impulse noise and background noise, impulse noise plus two frequency disturbances. In both cases, the code does improve performance considerably.

The impulse noise was simulated in accordance with the measurements reported in [3]. As frequency disturbances we assumed sine waves to be permanently present at the frequencies used to transmit the permutation code words.

CONCLUSIONS

We describe a new modulation/coding scheme that is capable of handling frequency disturbances like narrow band noise or impulse noise. For this we use the concept of M-FSK modulation combined with permutation codes. We modify the "simple" non-coherent demodulator in such a way that the demodulator output becomes multi-valued. The demodulator output is used in the decoding of the permutation code. It appears that background noise is of no importance up to a distance of 700 meters. The described modulation/coding scheme leads to an overall robust system.

References

Han Vinck & Haring

Coded 4-FSK Transmission at 4.800 Mbps, Threshold- and MAP-Detection
Background Noise.

Fig. 1.

4-FSK Transmission at 4.800 Mbps, Threshold Detection
Background Noise.

Fig. 2.

4-FSK Transmission at 4.800 Mbps, Threshold Detection
Background Noise.

Fig. 3.

4-FSK Transmission at 4.800 Mbps, Threshold Detection
Background Noise.

Fig. 4.

4-FSK Transmission at 4.800 Mbps, Threshold Detection
Background Noise, One Frequency Jammer.

Fig. 5.

4-FSK Transmission at 4.800 Mbps, Threshold Detection
Background Noise, Impulsive Noise.

Fig. 6.