Joint Channel and Impulsive Noise Estimation using Compressive Sensing for Powerline Communications

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Abstract—Channel estimation in conjunction with impulsive noise (IN) mitigation using compressive sensing is proposed for OFDM based power line communication (PLC) systems. Compressive sensing (CS) based IN mitigation algorithms use silent-pilots (or zero-subcarriers) to estimate and cancel the IN at receiver. To estimate the channel and IN jointly, non-zero pilots have to be used. However, the usage of non-zero pilots degrades the IN estimation due to channel frequency response terms appearing at the pilots’ positions. Therefore, we design a method which estimates the channel impulse response using CS, removes the channel frequency response terms appearing at the pilots’ positions and then estimates the IN using CS. The performance evaluation indicates that the proposed method improves the overall Bit Error rate (BER) of 1/2-rate coded-OFDM system to almost 4 dB as compared to the ‘Least Square’ (LS) method.

Index Terms—impulsive noise; compressive sensing; power line communication, impulsive noise mitigation, channel estimation, joint channel and impulsive noise estimation

I. INTRODUCTION

Powerline communication (PLC) has seen increased popularity from research as well as industrial community over the recent years. Due to its versatile nature which uses already built power cables for data communications, PLC provides cost effective and alternative solutions to in-house high data rate requirements. However, the PLC technology still confronts stiff obstacles in the fields of channel estimation and impulsive noise mitigation [1][2]. Traditionally, power lines have been considered as poor medium for high speed data transmission due to the presence of highly volatile impulsive noise and channel attenuations. Therefore most of the PLC applications in past never went beyond low rate transmission i.e. home automation/control, automated meter reading and premises security etc. But recent developments especially OFDM usage and impulsive noise mitigation made it possible to realize systems capable of transmitting high data rate, e.g. High Definition (HD) video and broadband communications, over in-house powerline networks[3], [4], [5].

Our recent experimental results indicate that the main cause of IN occurring on PLC network is the switching transients of various electrical appliances connected to mains network [6]. When these high power and burst INs occur, it is extremely difficult to maintain reliable communication link [2], [6].

Nevertheless, quest is on to improve the mitigation of IN and overall system performance. In this paper, we use a compressive sensing (CS) algorithm to jointly reconstruct the IN and channel impulse response (CIR) for OFDM based power line communications.

II. JOINT CHANNEL AND IN ESTIMATION FOR AN OFDM BASED PLC SYSTEM

Consider an OFDM based PLC system with Binary Phase Shift Keying (BPSK) modulation — as shown in Fig. 1 — with total number of subcarriers, pilot-subcarriers and data subcarriers in the OFDM frame denoted by $N$, $M$ and $N-M$ respectively. The pilots are distributed across the frame with randomly chosen indices[7] [8]. After FFT at receiver, the received OFDM can be shown in vector form as

$$\hat{x} = \hat{S}h + \hat{u} \quad (1)$$

The term $\hat{u} = \hat{e} + \hat{v}$ in (1) has the impulsive noise ($\hat{e}$) and white noise ($\hat{v}$) in frequency domain. The impulsive noise ($e_n$) is the product of high power Gaussian noise ($g_n$) and Bernoulli process ($b_n$) of zeros and ones i.e. $e_n = g_n b_n$ — with $\text{prob}(b_n = 1) = p = 0.01$. The variance ($\sigma_g$) of $g_n$ has been chosen to be 50 times the variance ($\sigma_v$) of $v_n$ i.e. $\sigma_g = 50\sigma_v$ to model the high power IN. Signal to Noise ratio
(SNR) has been defined as $SNR = \frac{S}{\sigma^2} = \frac{E_b}{N_0}$. Where, $E_b$ denotes the bit energy to noise density ratio. The PLC channel $h$ is generally considered as a network of power lines where joints in the network cause reflections to the transmitted signal and therefore a multipath model is used [9]. The channel is assumed to be invariant during the duration of one OFDM frame.

The diagonal elements of $\hat{S}$ in (1) denote the transmitted data frame in frequency domain which also contains $M$ pilots distributed across the frame and their positions are assumed to be known at the receiver. By sampling (1) at only these $M$ pilots positions, we have:

$$\hat{x}_M = \hat{s}_M \cdot (\sqrt{N}F_M h_N) + F_M e + \tilde{v}_M$$  \hspace{1cm} (2)

where, $\hat{s}_M$ and $\hat{h}_M$ denote vector containing pilot-symbols and channel frequency-response at pilots’ positions. The small dot “$\cdot$” in (2) denotes the element-to-element multiplication between vectors $\hat{s}_M$ and $\hat{h}_M$. In conventional CS based IN mitigation algorithms [8][10][11], silent-pilots are used for IN reconstruction i.e. $\hat{s}_M = 0$ which results in the disappearance of channel term $\hat{h}_M$. Consequently, silent-pilots cannot be used for channel estimation. On the other hand, if we use non-zero pilots in (2) i.e. $\hat{s}_M \neq 0$, we get the partial channel frequency response added into the overall noise term at pilots positions as:

$$\tilde{x}_M = \hat{s}_M \cdot (\sqrt{N}F_M h_N) + F_M e + \tilde{v}_M$$  \hspace{1cm} (3)

where, $h_N = [h_0, h_1, ..., h_{L-1}, 0, 0, 0]$ is a zero-padded vector of length $N$ with first $L$ samples representing the impulse response of the channel. Whereas, $F_M$ is Fast Fourier Transform (FFT) matrix $(F_{m,n} = \frac{1}{\sqrt{N}}e^{-j\frac{2\pi}{N}(m-1)(n-1)}$, $m, n = 0, 1, ..., N - 1$) sampled at only $M$ rows which correspond to pilots positions. In (3), we have a system where there are two sparse vectors (i.e. $h_N$ and $e$) jointly sampled by the partial FFT matrices and also contaminated by white noise. Here, we apply the LASSO algorithm [12] as:

$$h^S_N = \arg \min_{h_N} \frac{1}{2} \left\| \frac{\hat{x}_M}{\hat{s}_M} - \sqrt{N}F_M h_N \right\|_2^2 + \gamma \parallel h_N \parallel_1$$  \hspace{1cm} (4)

With the assumption that the supports of $h_N$ and $e$ are disjoint and we have sufficient number of measurements (pilots) i.e. $M \geq (|\text{supp}(h_N)| + |\text{supp}(e)|)C\mu \log(N)$, the LASSO algorithm reconstructs $h_N$ and $e$ distinctively in time-domain. Due to different scalings of $F_M$ in (3), we retain the first $L$ samples which correspond to channel impulse response and zero the rest by:

$$h^CS_N = \begin{cases} h^S_{L,i}, & i = 0, 1, ..., L - 1 \\ 0, & i = L, ..., N - 1. \end{cases}$$  \hspace{1cm} (5)

The zeroing of last $N - L$ samples of $h^CS_N$ ensures that the channel impulse response is free from IN contaminations and thereby could be used to remove the channel frequency-response effects from the pilots as:

$$\tilde{x}_M = \hat{s}_M \cdot (\sqrt{N}F_M h^CS_N)$$  \hspace{1cm} (6)

The crude IN estimate is then obtained by:

$$e^{CS} = \arg \min_{e} \frac{1}{2} \left\| \tilde{x}_M - F_M e \right\|_2^2 + \gamma \parallel e \parallel_1$$  \hspace{1cm} (7)

In (4) and (7), $\gamma$ is the regularization parameter and is optimized to 0.001 by hit-and-trail method [11]. The IN support estimation is done by keeping the largest $M_T/2$ samples of $e^{CS}$ with their indices set denoted by $\hat{I}$. The purpose of selecting the largest $M_T/2$ samples is to force the bound $M_T \geq TC\mu \log(N)$ to approximate to $M_T \geq T \times 2$ ($M_T$ can be considered as the number of pilots sufficient for accurate IN reconstruction only). After estimating the $\text{supp}$ (i.e. $\hat{I}$), the IN amplitudes are optimized by Least Square (LS) method as:

$$e^{CS} = \begin{cases} (F_M^H F_M)^{-1} F_M^H \tilde{x}_{M,\hat{I}}, & \text{on } \hat{I} \\ 0, & \text{elsewhere.} \end{cases}$$  \hspace{1cm} (8)

The resultant optimized IN estimate $e^{CS}$ is subtracted from the received OFDM $x$ frame as shown in fig 1.

### III. PERFORMANCE EVALUATIONS AND DISCUSSION

A convolutional coded OFDM system with coding rate $R = 1/2$ and proposed scheme has been evaluated in terms of
Fig. 3: MSE comparison of the ‘Least Square’ and ‘Compressive Sensing’ based channel estimations

Mean Square Error (MSE) and Bit Error Rate (BER) performances. The system has total number of subcarriers $N = 256$ with $M = 32$ pilots in each frame. With the probability of IN $\text{prob}(b_n = 1) = 0.01$ and channel taps $L = 5$, the number of pilots allocated for channel estimation $M_L = 10$ and IN reconstruction $M_T = 22$ are sufficiently large to be used with compressive sensing [13]. Two channel estimates i.e. $\hat{h}_N^{\text{LS}}$ (Least Square) and $\hat{h}_N^{\text{CS}}$ (Compressive Sensing) have been tested. Fig 3 shows the MSE comparison of LS and CS schemes in the presence and absence of impulsive noise in first $L$ samples of $h_N$. Although the CS based estimator suffers badly when IN occurs in first $L$ samples, its overall performance in terms of MSE is still better than that of LS based estimator. The BER simulations with estimated channel equalizations using LS and CS schemes and IN mitigation using CS are shown in fig 4 whereby the effects of channel estimate on BER are shown. It is observed from the figure that the subtraction of CS channel estimate $\hat{h}_N^{\text{CS}}$ in (6) and subsequent equalization by $\hat{h}_N^{\text{CS}}$ performs better than LS channel estimate.

IV. CONCLUSION

In this paper, we have presented a method to use same pilots set for channel estimation as well as IN reconstruction and cancellation using CS. Additionally, we also compare the channel estimation accuracy of least square (LS) method and compressive sensing (CS) method in the presence of impulsive noise whereby we observe that the performance of LS based method is not compareably good due to its failure to accurately reconstruct sparse signals (i.e. channel impulse response and IN). The BER simulations of 1/2 rate coded OFDM system indicate that the proposed CS based method provides the improvement of almost 4 dB in terms of $E_b/N_o$ at the BER of $10^{-5}$ as compared to LS based method. The proposed algorithm is suitable for the environment where the channel and IN change significantly over time like powerline communication systems.

REFERENCES